3.13

5. The Pyramid of Kukulcán at Chichén Itzá in Mexico (shown here) is a square pyramid. To the nearest foot, its base has side length 180 feet. Its height is 98 feet.

   a. What is the slant height of a square pyramid with base length 180 ft and height 98 ft?

   b. What is the lateral surface area of such a pyramid?

   c. If the pyramid is made from blocks 1 ft by 1 ft by 3 ft, estimate how many blocks are on the outside of the pyramid. Assume that the side of a block facing outward measures 1 ft by 3 ft.

6. Write a formula you can use to find the total surface area of a square pyramid if you know the length of a side of the base square and each of the following.

   a. the slant height   b. the height

   c. the length of each edge

7. a. The total surface area of a right rectangular prism is 119 square feet. Its base measures 5 feet by 8 feet. What is its height?

   b. Another right rectangular prism has total surface area 160 cm^2. It has a square base and is twice as tall as it is wide. What are its dimensions?

   c. **Take It Further** A right, regular hexagonal prism has height 4 in. Its base has side length 1 in. What is its total surface area?

8. a. The total surface area of a square pyramid is 84 m^2. It has a slant height of 4 m. How wide is its base?

   b. Another square pyramid has total surface area 160 cm^2. It has a slant height equal to twice the side length of its base. What are its dimensions?

   c. **Take It Further** A hexagonal pyramid has height 4 in. Its base has side length 1 in. What is its total surface area?

9. Write a formula for the total surface area of each given solid. Let s be the length of an edge.

   a. tetrahedron   b. cube

   c. octahedron   d. icosahedron

10. **Standardized Test Prep** What is the lateral surface area of a right square pyramid with base edge length 12 cm and height 8 cm?

    A. 192 cm^2 B. 240 cm^2 C. 346.13 cm^2 D. 384 cm^2

An octahedron has 8 triangular faces. An icosahedron has 20 triangular faces.

3.14

4. Find the lateral and total surface areas for the solids described.

   a. a right cylinder: radius 7 ft, height 12 ft

   b. a right cylinder: radius 12 ft, height 7 ft

   c. a right cone: diameter 12 cm, slant height 13 cm

   d. a right cone: diameter 13 cm, slant height 12 cm

5. You can make two different cylinders by taping together opposite edges of a standard $8\frac{1}{2}$ in.-by-11 in. sheet of paper. (You tape together either the two short edges or the two long edges.) For each cylinder, find the radius, height, lateral surface area, and total surface area (including bases).

6. A cone has base radius 15 cm and height 20 cm. A cylinder has the same height and the same lateral surface area as the cone. What is the radius of the cylinder?

7. A cylinder has base radius 10 cm and height 20 cm. A cone has a base congruent to that of the cylinder and the same lateral surface area as the cylinder. What is the height of the cone?

8. **What's Wrong Here?** Tony and Sasha were making up problems so they could practice finding surface area. They made up one problem that asked for the lateral surface area of a cone with radius 13 cm and slant height 5 cm.

   a. Sasha said, "Hey! Wait a minute! This cone isn't possible!" Draw a picture to show why.

   b. Tony said, "You're right. But I think we *can* have a cone with radius 13 cm and height 5 cm." Find the slant height of this cone. Then find its lateral surface area.

   c. What does this exercise tell you about how the slant height and the radius of a cone are related?

9. Standardized Test Prep What is the lateral surface area of an ice cream cone with diameter 3 in. and height 4 in.?

    A. 18.85 in.^{2} **B.** 20.13 in.^{2} **C.** 40.26 in.^{2} **D.** 47.12 in.^{2}

Cut a cone parallel to its base to get two pieces as in the diagram. The top piece is a new, smaller cone. The bottom piece is a **frustum.**
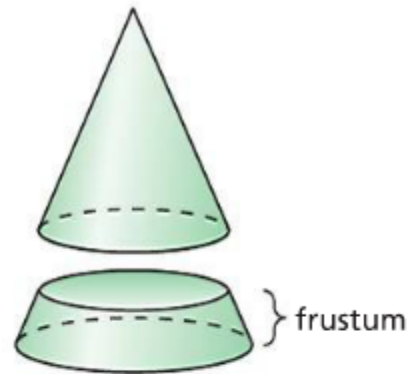
10. Take It Further A frustum has bottom radius 12 cm, top radius 9 cm, and height 4 cm.

   a. What is the slant height of the frustum?

   b. What was the slant height of the original cone before the cut? (*Hint:* Draw a cross section of the original cone. Think about the slope of a line.)

   c. What is the lateral surface area of the smaller cone formed by the cut?

   d. What is the lateral surface area of the frustum?

   e. What is the total surface area of the frustum?

11. Take It Further Many different frustums have a bottom radius of 13 cm and a slant height of 5 cm. The radius of the top of these frustums must fall within a certain range of values.

   a. How small can the radius of the top circle be?

   b. How large can the radius of the top circle be? (Remember that even though the bottom circle has radius 13 cm, that does not necessarily mean that it is the larger circular end of the frustum.)

   c. Think of a frustum with its top circle halfway between the two extremes. There is a special name for this frustum. What is it?

12. Take It Further If you had a frustum with one of the circles having radius 0, what else could you call this shape?

4. You have some apple cider in a cylindrical pot that has a radius of 4 in. You pour cider from the pot into a cylindrical cup that has a radius of $1\frac{1}{2}$ in. until the cider in the cup is 4 in. deep. How much did the level of cider in the pot go down?

5. A square prism and a cylinder have the same volume. The side length of the base of the prism is the same as the radius of the cylinder. The height of the prism is 7 cm. What is the height of the cylinder?

6. A factory has three cylindrical chimneys to release waste gases. Each chimney is 150 feet tall. The base of a chimney has a diameter of 15 feet. A can of paint can cover 100 square feet of surface. How many cans of paint are needed to paint the outsides of all three chimneys?

7. The factory in Exercise 6 needs to install filters inside the chimneys. The filter completely fills the interior of each chimney. The thickness of each chimney wall is $1\frac{1}{2}$ feet. How many cubic feet of filter are needed?

8. Sean is making a replica of the Washington Monument for history class. He decides to simplify the structure so that the replica resembles a square prism on the bottom and a square pyramid on the top. Sean's measurements are listed below. What is the volume of Sean's replica?

   - The total height of the replica is 24 inches.
   - The height of the pyramid at the top is 4 inches.
   - The length of one side of the square base of the replica is 3 inches.

9. **Standardized Test Prep** Chef Jasper's signature dessert is an individual carrot cake in the form of a right circular cone. The diameter of the cone is 3 in. and the height is 4 in. What is the approximate volume of carrot cake in each dessert?

   A. 9.425 in.^{3} **B.** 12 in.^{3} **C.** 28.27 in.^{3} **D.** 37.70 in.^{3}

10. **Take It Further** Margherite wants to make a more accurate replica of the Washington Monument. She determined the measurements as follows.

    - The height of her replica is 22 inches.

    - The length of one side of the square at the base of her replica is 2 inches.

    - The length of one side of the square at the base of the pyramid at the top is $1\frac{3}{8}$ inches.

    - The height of the pyramid is 2 inches.

    Margherite has one quart of plaster. Does she have enough to make the model? Explain.

11. Terry wants to find out how much liquid a paper cup will hold. He measures the cup to be 4 inches high. The diameter of the bottom of the cup is 2 inches. The diameter of the top is 3 inches. How many fluid ounces of liquid can the cup hold?

12. **Take It Further** A tall cone has radius 5 m and height 9 m. A short cone has radius 9 m and height 5 m.

    a. Find the volume of each cone. Which has a greater volume, the tall cone or the short cone?

    b. Cut another cone off of the tip of the cone that has the greater volume. Your goal is to leave a frustum that has the same volume as the smaller of the two original cones. What must be the height of the cone you cut off?

Remember...

1 gal = 231 in.^{3}

1 gal = 128 fl oz

1 gal = 4 qt