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4. **Standardized Test Prep** Which of the following statements is NOT true about a quadrilateral in a plane?
- A. Except for its endpoints, all the points on at least one of a quadrilateral's diagonals lie in the interior of the quadrilateral.
 - B. At least one diagonal of a quadrilateral divides the quadrilateral into two triangles.
 - C. The sum of the measures of the interior angles of a quadrilateral is 360° .
 - D. The four vertices of every quadrilateral lie on a unique circle.

Review the following triangle theorems. Then complete Exercises 5–8.

- The sum of the measures of the angles of a triangle is 180° .
 - The sum of the lengths of any two sides of a triangle is greater than the length of the third side.
5. The sum of the measures of the angles of a quadrilateral is always 360° . Explain why this is true.
6. Is the sum of the measures of the angles of a self-intersecting quadrilateral always 360° ? Explain.
7. Is the sum of the measures of the angles of a skew quadrilateral always 360° ? Explain.

8. **Take It Further** Prove that the sum of the lengths of any three sides of a quadrilateral is greater than the length of the fourth side. Determine whether your proof works for each type of quadrilateral listed below.

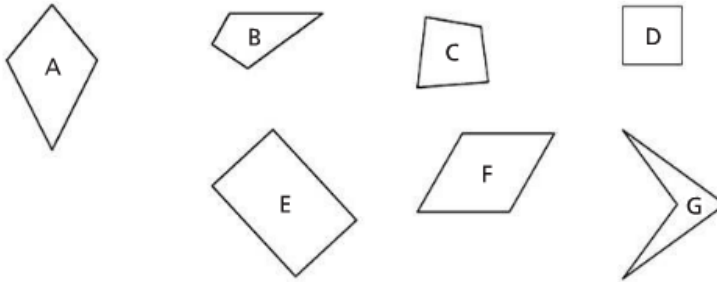
- concave
- self-intersecting
- skew

If your proof does not work for one or more types of the quadrilaterals listed above, can you write a different proof that will hold? Or does the proof fail because the property is not a characteristic of that type of quadrilateral?

How would you define the angles of a self-intersecting quadrilateral?

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12. Which of the following figures appear to be kites? Explain.



For Exercises 13–17, complete each sentence with *always*, *sometimes*, or *never* to make the statement true.

13. The sum of the measures of the angles of a kite is ? 360° .
 14. The diagonals of a kite are ? perpendicular.
 15. A kite ? has two congruent angles. 16. A kite ? has a right angle.
 17. One diagonal of a kite ? bisects one of its angles.
 18. **Standardized Test Prep** Which of the following statements is always true?
 I. A kite has at least one pair of congruent adjacent sides.
 II. A kite has at least one pair of congruent opposite angles.
 III. The diagonals of an isosceles trapezoid are congruent.
 A. I only B. I and II only C. I and III only D. I, II, and III
 19. Prove that the symmetry diagonal of a kite bisects two angles of the kite.
 20. Prove that the diagonals of a kite are perpendicular.

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For Exercises 9–28, complete each sentence with *always*, *sometimes*, or *never* to make the statement true.

9. A parallelogram ? has two congruent sides.
 10. A parallelogram ? has three congruent sides.
 11. A parallelogram ? has exactly three congruent sides.
 12. A parallelogram ? has four congruent sides.
 13. A parallelogram ? has congruent diagonals.
 14. A quadrilateral with congruent diagonals is ? a parallelogram.

Homework: Investigation 2D

Name: _____

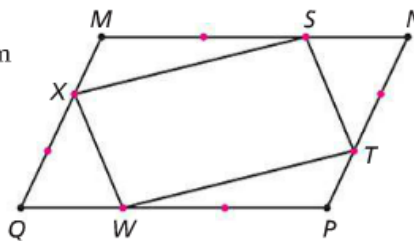
15. If one diagonal of a quadrilateral divides it into two congruent triangles, then the quadrilateral is ? a parallelogram.
16. If two consecutive angles of a quadrilateral are supplementary, then the quadrilateral is ? a parallelogram.
17. A quadrilateral with one right angle is ? a parallelogram.
18. A quadrilateral with exactly one right angle is ? a parallelogram.
19. A quadrilateral with two right angles is ? a parallelogram.
20. A quadrilateral with exactly two right angles is ? a parallelogram.
21. A quadrilateral with exactly two right angles opposite each other is ? a parallelogram.
22. A quadrilateral with three right angles is ? a parallelogram.
23. A quadrilateral with diagonals that bisect each other is ? a parallelogram.
24. If the longer diagonal of a quadrilateral bisects the shorter diagonal, then the quadrilateral is ? a parallelogram.
25. A quadrilateral with two congruent sides is ? a parallelogram.
26. A quadrilateral with three congruent sides is ? a parallelogram.
27. A quadrilateral with exactly three congruent sides is ? a parallelogram.
28. A quadrilateral with four congruent sides is ? a parallelogram.

Hint: Can a quadrilateral have exactly three right angles?

29. **Standardized Test Prep** Which of the following statements is NOT true?

- A. Every parallelogram has at least one line of symmetry.
- B. The diagonals of a parallelogram always bisect each other.
- C. Opposite angles of a parallelogram are congruent.
- D. Consecutive angles of a parallelogram are supplementary.

30. **Take It Further** In the figure at the right, the sides of parallelogram $MNPQ$ are trisected. Four of the trisection points form quadrilateral $STWX$.



- a. List some facts that you can prove about quadrilateral $STWX$.
- b. What type of quadrilateral is $STWX$? Prove your conjecture.
- c. Suppose you draw \overline{SW} and \overline{TX} . What can you say about these two segments?

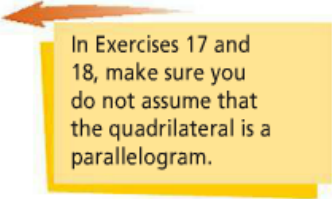
A segment is **trisected** if it is divided into three congruent parts.

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14. **Standardized Test Prep** Which of the following statements is NOT true?
- A. A square is different from a rhombus because a square has four congruent sides.
 - B. A rectangle is a parallelogram with four congruent angles.
 - C. A square is a rectangle with four congruent sides.
 - D. A rhombus is a parallelogram with four congruent sides.
15. Prove that a parallelogram with one right angle is a rectangle.
16. Prove that if the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle.
17. What type of quadrilateral do you form when you connect the midpoints of a rectangle's sides? Prove your conjecture.

For Exercises 18–23, prove each statement.

- 18. If a quadrilateral has four congruent sides, then it is a rhombus.
 - 19. Either diagonal of a rhombus divides the rhombus into two isosceles triangles.
 - 20. The diagonals of a rhombus bisect the vertex angles of the rhombus.
 - 21. The diagonals of a rhombus are perpendicular.
 - 22. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.
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- 23. If one diagonal of a parallelogram bisects two opposite angles of the parallelogram, then the parallelogram is a rhombus.
 - 24. Draw a square and connect the midpoints of its sides. Prove that the figure formed is also a square.



In Exercises 17 and 18, make sure you do not assume that the quadrilateral is a parallelogram.

Habits of Mind**Think it through.**

To prove that a quadrilateral is a square, can you just prove that its four sides are congruent?