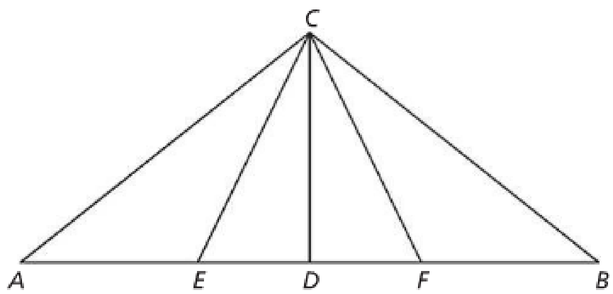
Homework Investigation 2C **Name:**

2.10

On Your Own

5. Standardized Test Prep In the figure below, $\triangle ABC$ is an isosceles triangle with $\overline{AC} \cong \overline{BC}$. $\overline{CD}$ is a median of $\triangle ABC$. $\overline{CE}$ bisects $\angle ACD$. $\overline{CF}$ bisects $\angle BCD$. Which of the following is a correct way to prove that $\triangle ACE \cong \triangle BCF$?



A. It is *not* possible to prove that $\triangle ACE \cong \triangle BCF$ based on the information given.

B. $\overline{AD} \cong \overline{BD}$, since D is the midpoint of $\overline{AB}$. So $\triangle ACD \cong \triangle BCD$ by SSS. This implies $\angle A \cong \angle B$ and $\angle ACD \cong \angle BCD$ by CPCTC. So $m\angle ACD = m\angle BCD$. Because $\overline{CE}$ and $\overline{CF}$ are angle bisectors, $m\angle BCF = \frac{1}{2}m\angle BCD = \frac{1}{2}m\angle ACD = m\angle ACE$. So $\angle BCF \cong \angle ACE$. Then $\triangle ACE \cong \triangle BCF$ by ASA.

C. Point D is the midpoint of $\overline{AB}$, so $\overline{AD} \cong \overline{BD}$. Then $\triangle ACD \cong \triangle BCD$ by SSS and $\overline{AE} \cong \overline{FB}$ by CPCTC. This implies $\triangle ACE \cong \triangle BCF$ by SAS.

D. Since $\overline{CD}$ is a median of $\triangle ABC$, D is the midpoint of $\overline{AB}$. So $\overline{AD} \cong \overline{BD}$. Because $\overline{CE}$ and $\overline{CF}$ are angle bisectors, they divide $\overline{AD}$ and $\overline{BD}$, respectively, into two congruent segments. So $\overline{AE} \cong \overline{ED}$ and $\overline{BF} \cong \overline{FD}$. Because $\triangle ABC$ is an isosceles triangle with $\angle A \cong \angle B$, the sides opposite those angles are congruent. So $\overline{AC} \cong \overline{BC}$. Since $\overline{AE} \cong \overline{BF}$, $\triangle ACE \cong \triangle BCF$ by SAS.

Remember...

An **isosceles triangle** is a triangle with at least two congruent sides. You call the congruent sides **legs.** The **vertex angle** is the angle formed by the two legs. You call the third side the **base.** The angles on either side of the base are the **base angles.**

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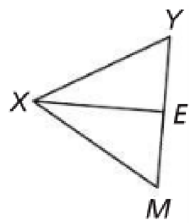
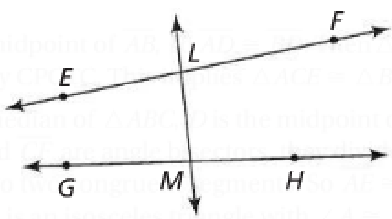
6. Draw an isosceles triangle and the bisector of its vertex angle. Prove that the two smaller triangles formed are congruent.

7. In $\triangle XMY$, $\overline{XE}$ is a median, and $\overline{XY} \cong \overline{XM}$. The bulleted list below is a sketch of a proof that $\triangle XEM \cong \triangle XEY$. Study the list. Then write the proof in either two-column, paragraph, or outline style.



- $\overline{XE}$ is a median, so E is the midpoint of $\overline{MY}$. Then $\overline{ME} \cong \overline{EY}$.
- $\overline{XY} \cong \overline{XM}$ is given.
- The two triangles share $\overline{XE}$.
- The triangles have three pairs of congruent sides.

8. What's Wrong Here? Below is a proof that shows that any two lines are parallel. Explain what is wrong with the proof.



Suppose $\overleftrightarrow{EF}$ and $\overleftrightarrow{GH}$ are lines. Draw transversal $\overleftrightarrow{LM}$.

$\because \angle ELM$ and $\angle HML$ are alternate interior angles.

$\therefore \angle ELM \cong \angle HML$

$\therefore \overleftrightarrow{EF} \parallel \overleftrightarrow{GH}$ (AIP Theorem)

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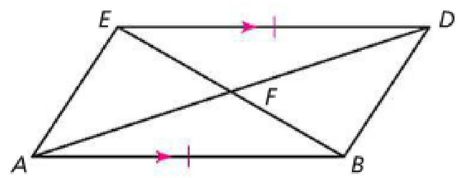


9. In the figure at the right, $\overleftrightarrow{AB} \parallel \overleftrightarrow{ED}$ and $\overline{AB} \cong \overline{ED}$.

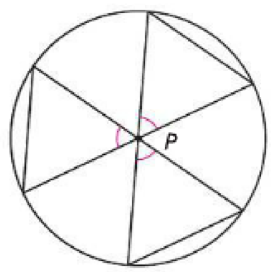
a. Timothy uses this information to prove that $\triangle ABF \cong \triangle DEF$. Explain why his paragraph proof is incorrect.

It is given that $\overleftrightarrow{AB} \parallel \overleftrightarrow{ED}$, so $\angle DEB \cong \angle ABE$ because parallel lines form congruent alternate interior angles with a transversal. And $\angle AFB \cong \angle DFE$ because they are vertical angles, and vertical angles are congruent. It is also given that $\overline{AB} \cong \overline{ED}$, so $\triangle ABF \cong \triangle DEF$ by ASA.

b. Is it possible to prove that $\triangle ABF \cong \triangle DEF$? If so, write a correct proof.

10. To show that a statement is true, mathematicians require deductive proof. Explain how you can convince someone that the statement "All horses are the same color" is *not* true.

11. Use circle *P* below to prove that the three triangles are congruent. Point *P* is the center of the circle.



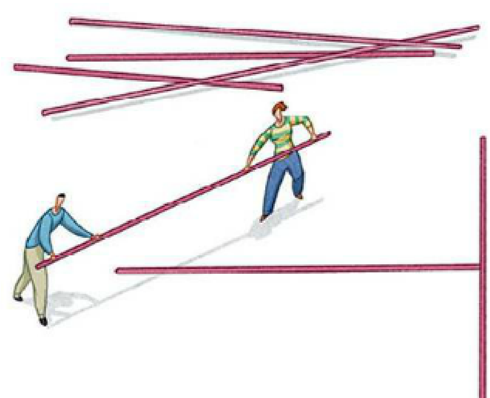
2.11

On Your Own

3. **Standardized Test Prep** Which is the hypothesis of the following statement? If two angles are congruent, then they have the same measure.

A. Two angles are congruent. **B.** They have the same measure.

C. Two angles are not congruent. **D.** They do not have the same measure.



For Exercises 4–9, draw a picture that illustrates the hypothesis. Then determine whether the statement is true. If a statement is true, give a proof. If a statement is not true, give a counterexample.

4. In a plane, two lines that are perpendicular to the same line are parallel to each other.

5. A line that bisects an angle of a triangle also bisects the side that is opposite the angle.

6. Equilateral quadrilaterals are **equiangular** (all angles congruent).

7. If a triangle has two congruent angles, it is isosceles.

8. Equiangular triangles are equilateral.

9. Equiangular quadrilaterals are equilateral.

Homework Investigation 2C **Name:**

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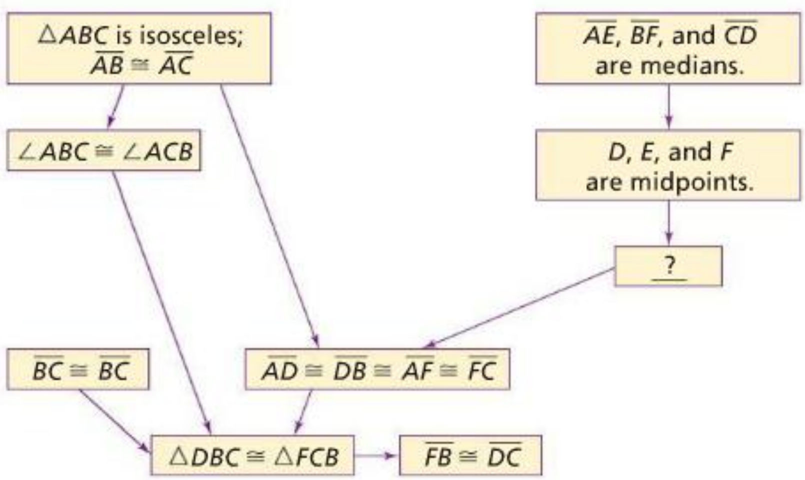
On Your Own

4. Standardized Test Prep $\triangle ABC$ is an isosceles triangle with $\overline{AB} \cong \overline{AC}$. $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are medians.

The flowchart below outlines a proof that shows that $\overline{FB} \cong \overline{DC}$. Which of the statements below best completes the flowchart?



A. $\angle ABF \cong \angle FBC$

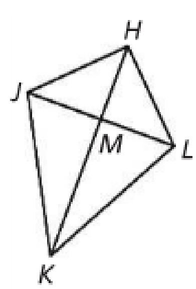
B. $\overline{AD} \cong \overline{DB}$; $\overline{AF} \cong \overline{FC}$; $\overline{BE} \cong \overline{EC}$

C. $\overline{AE} \cong \overline{AE}$

D. $\triangle ABE \cong \triangle ACE$

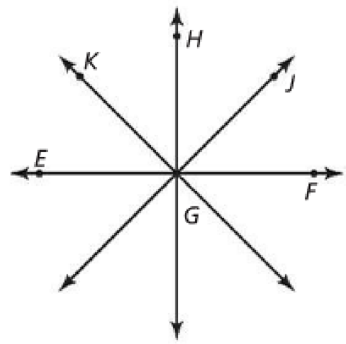
For Exercise 5, use the visual scan strategy to analyze the proof. Copy the figure onto a separate piece of paper, and mark the given information. Then write an outline for the proof.

5. Given $\overline{HJ} \cong \overline{HL}$ and $\overline{JK} \cong \overline{LK}$
Prove $\triangle HJM \cong \triangle HLM$

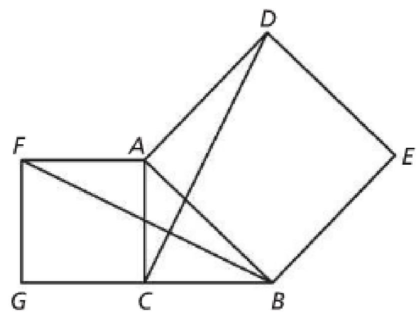


Homework Investigation 2C **Name:**

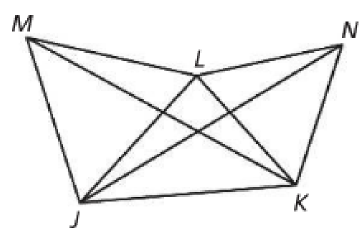
6. **Given** $\overleftrightarrow{GF} \perp \overleftrightarrow{GH}$ and $\overleftrightarrow{GJ} \perp \overleftrightarrow{GK}$
Prove $\angle JGF \cong \angle KGH$



7. **Given** $FACG$ and $DABE$ are squares.
Prove $\triangle FAB \cong \triangle CAD$



8. **Given** $\triangle LJM$ and $\triangle LKN$ are equilateral.
Prove $\overline{MK} \cong \overline{NJ}$



9. **Given** $\triangle QRT$ is an isosceles triangle. $\overline{QT} \cong \overline{TR}$. $\overline{VR}$ and $\overline{UQ}$ are medians.
Prove $VR = UQ$

10. **Given** Point P is on the perpendicular bisector of $\overline{LM}$.
Prove $PL = PM$

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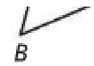
2.13



On Your Own

In Exercises 3–8, prove each statement. Use a reverse list to write each proof.

3. If a triangle is isosceles, then the medians from its legs to the vertices of its base angles are congruent.

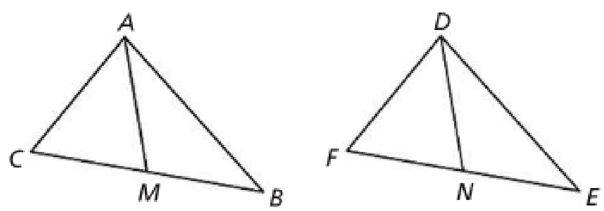
4. If a triangle is isosceles, then the bisectors of its base angles are congruent.

5. If two altitudes of a triangle are congruent, then the triangle is isosceles.

6. If a triangle is isosceles, then the altitudes drawn to its legs are congruent.

7. In isosceles triangle ABC, $\overline{AC} \cong \overline{BC}$. Point M is the midpoint of $\overline{AC}$. Point N is the midpoint of $\overline{CB}$. Prove that $\triangle CMN$ is an isosceles triangle.

8. **Take It Further** In the two triangles below, $\overline{AC} \cong \overline{DF}$ and $\overline{CB} \cong \overline{FE}$. $\overline{AM}$ and $\overline{DN}$ are congruent medians. Show that $\triangle ABC \cong \triangle DEF$.



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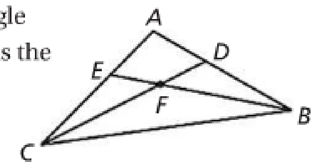
9. **Standardized Test Prep** Triangle ABC at the right is an isosceles triangle with $\overline{AB} \cong AC$ and $\angle ABC \cong \angle ACB$. $\overline{CD}$ is the bisector of $\angle ACB$. $\overline{BE}$ is the bisector of $\angle ABC$. Suppose you want to prove that $\overline{EF} \cong \overline{DF}$.



Which of the statements and reasons below complete the following reverse list?

Need $\overline{EF} \cong \overline{DF}$
Use CPCTC

Need I
Use I

Need II
Use II

Need III
Use III

Need $\triangle BCE \cong \triangle CBD$
Use ASA

Need $\overline{BC} \cong \overline{BC}$
Use The two triangles share the side.

Need $\angle ACD \cong \angle DCB \cong \angle ABE \cong \angle EBC$
Use definition of congruent angles

Need $m\angle ACD = m\angle DCB = m\angle ABE = m\angle EBC$
Use definition of angle bisector

Need $\overline{CD}$ is the bisector of $\angle ACB$. $\overline{BE}$ is the bisector of $\angle ABC$.
Use given

Need $\frac{1}{2} m\angle ABC = \frac{1}{2} m\angle ACB$
Use multiplication property of equality

Need $m\angle ABC = m\angle ACB$
Use definition of congruent angles

Need $\angle ABC \cong \angle ACB$
Use given

A. I. $\triangle BDF \cong \triangle CEF$; AAS
II. $\angle CFE \cong \angle BFD$; the Vertical Angles Theorem
III. $\overline{CE} \cong \overline{BD}$; CPCTC

B. I. $\triangle ADC \cong \triangle AEB$; SAS
II. $\overline{AB} \cong \overline{AC}$; given
III. $\overline{BE} \cong \overline{CD}$; CPCTC

C. I. $\triangle ADC \cong \triangle AEB$; AAS
II. $\overline{AB} \cong \overline{AC}$; given
III. $\angle A \cong \angle A$; the two triangles share this angle.

D. I. $\triangle BDF \cong \triangle CEF$; ASA
II. $\angle BDF \cong \angle CEF$; CPCTC
III. $\overline{BE} \cong \overline{CD}$; CPCTC

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On Your Own

7. Refer to the diagram for Exercise 6. Write one new statement, either about $\triangle ABC$ or about $\triangle ADC$ and $\triangle BDC$, that guarantees that all four of the statements in Exercise 6 are true.

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8. Describe the set of triangles that meets each of the following conditions.

a. The perpendicular bisector of exactly one side passes through the opposite vertex.

b. The perpendicular bisector of each of exactly two of the sides passes through the opposite vertex.

c. The perpendicular bisector of each of the sides passes through the opposite vertex.

You may find it helpful to construct some triangles that meet each condition and some triangles that do not. Explain what you find.

9. Use a cup or glass to trace a circle on a sheet of paper. Explain how to find the center of the circle.

10. **Standardized Test Prep** Brittany found a piece of broken pottery that looks like part of a circular dinner plate. She wants to determine the radius of the plate, but she has less than half of the plate. First, she traces the outline of the outer edge of the pottery on a sheet of paper. Then she draws two line segments with endpoints that are on different parts of the curve. Next, she constructs the perpendicular bisector of each line segment. She extends the perpendicular bisectors until they intersect. Finally, she measures the distance from the point of intersection of the perpendicular bisectors to a point on the curve. Why does this procedure guarantee that she has found the radius of the plate?

A. A line that is perpendicular to a tangent of a circle will always go through the center of the circle. Each bisected line segment is tangent to the circle.

B. The perpendicular bisector of a chord divides the arc associated with the chord into two congruent arcs. The length of each arc is equal to the radius of the circle.

C. A point on the perpendicular bisector of a segment is equidistant from the endpoints of the segment. The distance from the intersection of the perpendicular bisectors to a point on a circle is the radius of the circle.

D. The point of intersection of the perpendicular bisectors, along with the midpoints of the bisected segments, determine an isosceles triangle. The segment that connects the midpoints of bisected segments is the base of the isosceles triangle. The radius of the circle is the length of one of the legs of the isosceles triangle.

How can you make a model of the original plate if you find only this piece?

11. Describe an algorithm that you can use to construct a circle that passes through the vertices of a given triangle.

Remember...

What does the term *algorithm* mean?

12. A circular saw blade shattered. All you can find is a piece that looks like the figure at the right. Explain how to find the diameter of the blade so you can buy a new one.

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13. Show that if the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.

This test for right-triangle congruence is sometimes called "hypotenuse-leg" and is abbreviated HL.

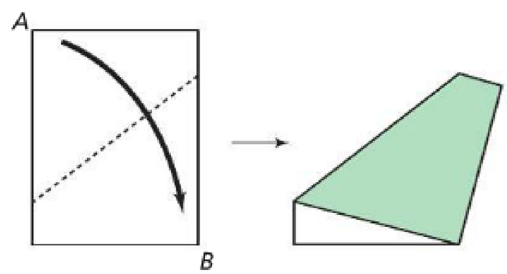
Habits of Mind

Be efficient. If you use geometry software, you can make one construction and just drag it around.

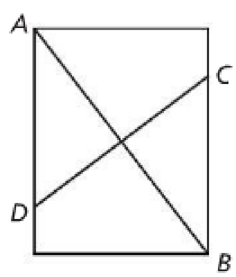
14. Draw several triangles. In each triangle, construct the perpendicular bisector of each side. Notice that the perpendicular bisectors in each triangle intersect in one point. Is this true for all triangles? Provide a proof or a counterexample.

15. Perform the paper-folding construction shown at the right. Start with a rectangular sheet of paper.

Fold *A* onto *B* and crease along the dotted line.

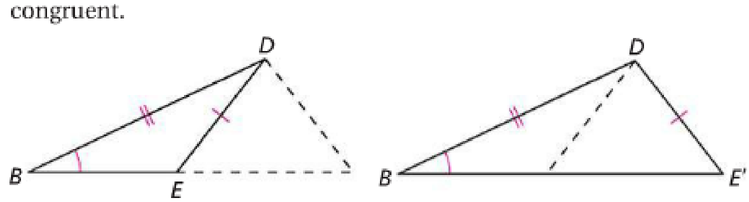


Unfold the paper. The creases should look like this.



Prove that any point on $\overline{CD}$ is the same distance from *A* as from *B*.

There is no SSA triangle postulate. Two sides and a nonincluded angle of one triangle can be congruent to two sides and the corresponding nonincluded angle of another triangle without the two triangles being congruent.



16. Take It Further Could there be an SSA postulate? Suppose you know that the nonincluded angle is the *largest* angle in each triangle. That is, suppose that the two sides and the largest nonincluded angle of one triangle are congruent to the corresponding two sides and largest angle of the other triangle. Can you conclude that the triangles are congruent? Explain.

If SSA does guarantee triangle congruence, it is a generalization of HL. Explain.

17. Take It Further And is there an SSa postulate? Suppose you know that the nonincluded angle is the *smallest* angle in each triangle. Can you conclude that the triangles are congruent? Explain.